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MEMO ON EXTRACTING $|V(cb)|$ AND $|V(ub)/V(cb)|$ FROM SEMILEPTONIC B DECAYS

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Abstract

Heavy Quark Expansions for semileptonic decays of beauty hadrons are briefly reviewed. I analyze how $|V(cb)|$ can be extracted from the semileptonic width of B mesons, the average semileptonic width of all weakly decaying beauty hadrons and from $B \to l\nu D^*$ at zero recoil. Special attention is paid to present theoretical uncertainties (including correlations among them) and on how to reduce them in the future. Finally I will comment on theoretical uncertainties in $|V(ub)/V(cb)|$.

Contents

1 Goal

I will concentrate on semileptonic transitions since I do not see any realistic hope to extract $|V(cb)|$ from nonleptonic decays with competitive accuracy. I provide here an Executive Summary stating the relevant results, their theoretical uncertainties and how the latter can be reduced in the future. The derivations can be found in reviews like [1, 2] and in the original papers listed there.

2 Theoretical Technologies

2.1 Fundamentals

Heavy Quark Theory is implemented through the operator product expansion (OPE) in the form of a heavy quark expansion (HQE). It allows to evaluate inclusive transition rates as an expansion in inverse powers of the heavy quark mass:

$$\Gamma(B \to l\nu X_q) = \frac{G_F^2 m_b^5}{192\pi^3}|V(qb)|^2 \left[c_3\langle B|O_3|B\rangle + c_5\frac{\langle B|O_5|B\rangle}{m_b^2} + c_6\frac{\langle B|O_6|B\rangle}{m_b^3} + \mathcal{O}(1/m_b^4)\right] \tag{1}$$

where the O_d denote local operators of (scale) dimension d. It is basically the same cast of operators – albeit with different weights – that appears in semileptonic, radiative and nonleptonic rates as well as distributions.

The coefficients c_d are calculated from short-distance physics; they contain the masses of the final state quarks from phase space etc. and powers of the strong coupling α_S. The hadronic expectation values (HEV) of the operators O_d encode the *nonperturbative* corrections. While we can identify these operators and their dimensions which then determine the power of $1/m_b$, in general we cannot (yet) compute their HEV's from first principles.

For the HEV of the leading operator $O_3 = \bar{b}b$ one has

$$\langle B|O_3|B\rangle/2M_B = 1 + \mathcal{O}(1/m_b^2) \; ; \tag{2}$$

it thus incorporates the parton model result which dominates asymptotically, i.e. for $m_b \to \infty$.

The most remarkable feature of Eq.(1) is the *absence* of a contribution in order $1/m_b$. *Such a term is anathema in the OPE since there is no relevant operator of dimension four* [1]! This has an important consequence:

- With the leading nonperturbative corrections emerging in order $1/m_b^2$, they can amount to no more than typically several percent in beauty decays. More specifically they *reduce* the semileptonic B width by close to 5 %. That also means that evaluating the HEV's with no more than moderate accuracy – say 20 % – already limits the overall uncertainties due to nonperturbative corrections to the 1% level.

Sum rules [4] serve as an important theoretical tool: they yield well-defined relations between basic parameters of HQE and provide insights into how quark-hadron duality comes about.

One warning should be stated explicitely: it would be *illegitimate* to merely replace *quark* masses in the OPE expressions by *hadron* masses. It can be shown [3] how sum rules lead to the emergence of quark phase space and the OPE nonperturbative corrections from the combination of hadronic phase space and bound state effects.

2.2 Determining the Size of Basic Parameters

[1] The operator $\bar{Q}\gamma \cdot \partial Q$ can be reduced to $m_Q \bar{Q} Q$ by the equation of motion.

2.2.1 Quark Masses

Since the semileptonic width depends on the fifth power of the heavy quark mass, great care has to be applied in treating the latter. Masses like other parameters in a quantum field theory are not constants, but vary with the energy scale μ at which they are evaluated: $m_b(\mu)$. In the limit $\mu \to 0$ the *pole* mass emerges; since – due to a so-called renormalon singularity – it suffers from an *irreducible* uncertainty $\sim \mathcal{O}(1/m_Q)$ that is *larger* than the nonperturbative contributions one is evaluating, it is in principle inadequate for our purposes here. Such problems are avoided if one chooses a scale $\mu \geq 0.5$ GeV which shields $m_b(\mu)$ against renormalon singularities. It turns out that the natural choice for this scale is $\mu \sim m_b/5 \sim 1$ GeV. The $\overline{\rm MS}$ mass is ill-suited for describing *decay* (though not production) processes since it treats scales below its value in an unphysical way. Instead one adopts a more natural definition for the low scale running mass which leads to a *linear* dependance on μ:

$$\frac{dm_b(\mu)}{d\mu} = -c_m \frac{\alpha_S(\mu)}{\pi} + ... \; ; \tag{3}$$

the number c_m specifies the concrete definition within this general class; reasonable choices are $c_m = 4/3$ or $16/9$.

Since $\mu \sim 1$ GeV, $m_b(\mu)$ can conveniently be inferred from $e^+e^- \to b\bar{b}$ very close to threshold. That way one arrives at (for $c_m = 16/9$)

$$m_b(1\text{ GeV}) = \begin{cases} 4.56 \pm 0.06 \text{ GeV} & [5] \\ 4.57 \pm 0.04 \text{ GeV} & [7] \\ 4.59 \pm 0.08 \text{ GeV} & [8] \end{cases} \tag{4}$$

The value of the mass difference $m_b - m_c$ can be inferred from the measured values of the spin averaged beauty and charm mesons:

$$\begin{aligned} m_b - m_c &= \langle M_B \rangle - \langle M_D \rangle + \mu_\pi^2 \left(\frac{1}{2m_c} - \frac{1}{2m_b} \right) + \mathcal{O}(1/m_{c,b}^2) \\ &\simeq 3.50\,\text{GeV}^2 + 40\,\text{MeV} \cdot \frac{\mu_\pi^2 - 0.5\,\text{GeV}^2}{0.1\,\text{GeV}^2} + \Delta M_2 \end{aligned} \tag{5}$$

where

$$\langle M_{B[D]} \rangle = \frac{M_{B[D]} + 3M_{B^*[D^*]}}{4} \; , \quad |\Delta M_2| \leq 0.02\,\text{GeV} \tag{6}$$

2.2.2 Hadronic Expectation Values

There are two dimension-five operators, namely the Lorentz invariant chromomagnetic operator $\bar{b}\frac{i}{2}g_s\sigma \cdot Gb$ and the kinetic energy operator $\bar{b}(i\vec{D})^2 b$ where $\vec{D}$ denotes the covariant derivative. The latter operator does not form a Lorentz scalar and therefore can enter only through the expansion of $\bar{b}b$.

- μ_G^2:

  The size of the chromomagnetic HEV can be estimated through the hyperfine splitting

  $$\mu_G^2 \equiv \frac{1}{2M_B}\langle B|\bar{b}\frac{i}{2}g_s\sigma\cdot Gb|B\rangle \simeq \frac{3}{4}\left(M_{B^*}^2 - M_B^2\right)\pm 20\% \simeq 0.36 \text{ GeV}^2 \pm 20\% \ , \quad (7)$$

  yet it cannot automatically be equated with it. For the equality holds only asymptotically, namely in the limit $m_b \to \infty$. At preasymptotic, i.e. finite values corrections of order $1/m_b$ (and higher) will arise. While it has been demonstrated in principle how those can be determined through the SV sum rules [1], this program has not been performed yet in a concrete way. This is a refinement that can be achieved in the future.

- μ_π^2:

  The quantity

  $$\mu_\pi^2 \equiv \frac{1}{2M_B}\langle B|\bar{b}(i\vec{D})^2 b|B\rangle \quad (8)$$

  is related to the average kinetic energy of the b quark moving inside the B meson.

  While its precise value is not known yet, we can state the following:

  - An analysis based on QCD sum rules yields [18]:

    $$\mu_\pi^2 \simeq (0.5 \pm 0.15)\ (\text{GeV})^2\ . \quad (9)$$

  - In the literature μ_π^2 is often equated with the HQET parameter λ_1 (or $-\lambda_1$). While the two quantities look superficially the same, they are *not* once quantum corrections are included. Those are essential for a self-consistent treatment.

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    $$\mu_\pi^2 \geq \mu_G^2 \quad (10)$$

    holds as a field theoretical inequality [2].

  - Since $\mu_\pi^2 = \langle|\vec{p}_b|^2\rangle$ one would be quite surprised if μ_π^2 exceeded the bound $\mu_G^2 \simeq (0.6 \text{ GeV})^2$ considerably.

  - Altogether a fairly conservative estimate is

    $$\mu_\pi^2 \simeq 0.5 \pm 0.2 \text{ GeV}^2 \quad (11)$$

  The remarks given above concerning preasymptotic corrections apply here as well.

[2] The dedicated reader should note that the exact definition of μ_π^2 employed here is not identical to that of Ref.[18].

2.3 On Theoretical Uncertainties

A few general comments on estimating uncertainties might be helpful:

- The *conditio sine qua non* for the validity of the OPE result is quark-hadron duality. While it is expected to hold in the decays of beauty hadrons for $m_b \to \infty$, there is a topical debate in the literature on its quantitative limitations for *finite* values of m_b when applied to *nonleptonic* transitions. Yet here we are dealing with semileptonic decays which present a smoother dynamical stage.

  These issues can be analyzed in the 't Hooft model – QCD in 1+1 dimensions with the number of colours $N_C \to \infty$ – which is an exactly soluble field theory; i.e., the spectrum of its boundstates and their wavefunctions are known. It automatically implements confinement. The validity of duality can be probed by comparing the OPE based result with the one obtained by summing over the hadronic resonances. Perfect matching has been found for semileptonic (as well as nonleptonic) widths. Furthermore duality violations can be modelled and were found to be suppressed by high powers of $1/m_Q$ [11, 19, 20].

- In stating a theoretical error for a quantity I mean that the real value can lie almost anywhere in this range with basically equal probability rather than follow a Gaussian distribution. Furthermore my message is that I would be quite surprised if the real value would fall outside this range. Maybe one could call that a 90 % confidence level, but I do not see any way to be more quantitative.

- One of the most intriguing features of HQE is that they allow us to express a host of observables – rates of nonleptonic, semileptonic and radiative decays, distributions etc. – in terms of a handful of basic quantities, namely quark masses and expectation values of the leading operators as sketched above. This means, though, that *correlations* exist between the predictions for different observables which have to be accounted for. However, this is not always done in the literature.

- The just mentioned feature of HQE can be exploited for important self-consistency checks by determining a basic quantity in two *systematically distinct* ways. If such a 'redundant' extraction provides a self-consistent value, control has been established over the theoretical uncertainty; otherwise the need for further analysis has been revealed.

- For the HQE to make numerical sense, one needs

  $$m_Q \gg \text{"typical" hadronic scales} \tag{12}$$

  This requirement is certainly satisfied for *beauty* quarks for any reasonable definition of "typical" hadronic scales. Yet for *charm* quarks it makes a difference

whether those are, say, 0.7 or 1 or 1.2 GeV. At present we cannot decide this isssue with certainty. I will return to this point later on and illustrate it through an example.

3 $\Gamma(B \to l\nu X_c)$

The basic expression is given by

$$\Gamma_{SL}(B) = \frac{G_F^2 m_b^5 |V(cb)|^2}{192\pi^3} \left[z_0 \left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b^2} \right) - 2 \left(1 - \frac{m_c^2}{m_b^2} \right)^4 \frac{\mu_G^2}{m_b^2} - \frac{2\alpha_S}{3\pi} z_0^{(1)} + ... \right] \tag{13}$$

where the omitted terms are higher-order perturbative corrections and/or power corrections; z_0, $z_0^{(1)}$ are *known* phase space factors depending on m_c^2/m_b^2. The leading power corrections are controlled by the HEV's μ_π^2 and μ_G^2. While the order α_S^2 contributions have not been stated explicitely in Eq.(13), they are relevant and have been included in the numerical analysis.

On the *conceptual* level the following has to be kept in mind:

- While the expansion in Eq.(13) is formulated in inverse powers of m_b, the true expansion parameter is provided by the energy release. This is easily expressed for two complementary cases: for light final state quarks – $m_c \to 0$ – the mass dependance is given by m_b^5, whereas for heavy ones – $m_c \to m_b$ – it is $(m_b - m_c)^5$ [3]. Intermediate scenarios can effectively be described in terms of powers of the difference $m_b - m_c$ and of m_b:

  $$\Gamma_{SL}(b) \propto G_F^2 (m_b - m_c)^{5-p} m_b^p$$

  with $p \sim 2$.

  It should be noted that the energy release is quite a bit larger for $b \to u$ than $b \to c$ transitions.

- Although I did not make it explicit in the expressions above, the matrix elements depend on a normalization scale μ. It emerges in the OPE through Wilson's prescription for separating long distance dynamics, which are lumped into the matrix elements, and short distance dynamics included in the c number coefficient functions of the operators in the OPE. The observable $\Gamma_{SL}(B)$ *in principle* cannot depend on this auxiliary scale since the μ dependance of the matrix elements obviously cancels against that of the coefficient functions

[3] HQE are expected to provide a reliable treatment for $b \to u$ as well as $b \to c$ since m_b as well as $m_b - m_c$ exceed ordinary hadronic scales $\bar{\Lambda} \sim \frac{1}{2}$ GeV.

in a complete calculation. *In practice* though one has to choose it judiciously for computational purposes: it has to satisfy

$$\Lambda_{QCD} \ll \mu \ll m_b \tag{14}$$

to enable us to evaluate both the perturbative corrections and the HEV's. In particular that means that the (running) quark masses that enter in the coefficient functions have to be evaluated at the same scale μ.

I use $\mu \sim 1$ GeV for the results stated below.

- The normalization scale μ should not be confused with the argument in the strong coupling α_S entering in the last term of Eq.(13), which is often denoted by μ as well; yet I will refer to it as $\tilde{\mu}$. While my μ reflects the proper normalization of the various operators including those generating the nonperturbative contributions, the $\alpha_S(\tilde{\mu})$ corrections are generated by the dynamics of high momenta of order $\tilde{\mu} \gg \mu$.

  It turns out that the main impact of the radiative corrections has been accounted for once the quark masses have been evaluated at μ as stated above. The dependance on $\tilde{\mu}$ then becomes a secondary one. [4]

Numerically one can state:

- The direct nonperturbative corrections of order $1/m_b^2$ are rather small and under control.

- The main challenge then consists of evaluating the *perturbative* expansion

$$\Gamma(b \to l\nu c) = \frac{G_F^2 m_b^5(\mu)}{192\pi^3} |V(cb)|^2 \left[a_0(m_c/m_b) + a_1(m_c/m_b)\frac{\alpha_S(\tilde{\mu})}{\pi} + a_2(m_c/m_b)\left(\frac{\alpha_S(\tilde{\mu})}{\pi}\right)^2 + \mathcal{O}(\alpha_S^3)\right] \tag{15}$$

  where the phase space corrections for $m_c \neq 0$ enter through the coefficients a_i.

- Any change in the normalization of the mass

$$m_b(\mu) \to m_b(\mu + \Delta\mu) = m_b(\mu) + \left[k_1 \frac{\alpha_S}{\pi} + k_2 \left(\frac{\alpha_S}{\pi}\right)^2 + ...\right] \Delta\mu \tag{16}$$

[4] Having settled the scale at which α_S should be evaluated one still has to determine its size at that scale. Extrapolating down from values for α_S obtained at high scales like the Z^0 mass might not be quite straightforward. It has been suggested – although not established – that the 'running' of the strong coupling is slowed down at lower scales due to nonperturbative effects.

greatly affects the perturbative coefficients a_i, in particular because of the high power with which m_b enters.

If a calculation yields a large value for the second order coefficient a_2, one certainly has to be concerned that a_3 etc. could be sizeable as well. While this could signal that the perturbative corrections introduce a considerable uncertainty, it could also mean that these large coefficients are an *artefact* of choosing an unnatural scale for evaluating m_b. It has been shown [3] that the latter is the case here. Adopting $\mu \sim m_b/5$ effectively sums a sequence of higher-order contributions; the remaining coefficients are of order unity making the uncertainty due to perturbative corrections small.

With the size of the quark masses of obvious importance they have to be defined properly and treated carefully, as mentioned above.

Equating the expression in Eq.(13) with the measured width one arrives at

$$\begin{aligned} |V(cb)|_{\Gamma_{SL}(B)} &= 0.0411\sqrt{\frac{BR(B\to l\nu X)}{0.105}}\sqrt{\frac{1.55\,\mathrm{ps}}{\tau(B)}} \\ &\times \left(1 - 0.025\cdot\frac{\mu_\pi^2 - 0.5\,\mathrm{GeV}^2}{0.2\,\mathrm{GeV}^2}\right) \\ &\times \left(1 \pm 0.01|_{m_b} \pm 0.01|_{pert} \pm 0.015|_{1/m_Q^3}\right) \end{aligned} \tag{17}$$

where the second and third lines refer to the theoretical uncertainties. I have taken into account here the correlations alluded to in the introduction. For the coefficient in the bracket in the second line reflects the fact that the size of μ_π^2 affects also the value of $m_b - m_c$, see Eq.(5). The *remaining* sensitivity to m_b enters through δm_b, where $\delta m_b/m_b \simeq 0.01$ has been assumed. The last term finally represents an estimate of power corrections beyond order $1/m_b^2$, including possible deviations from quark-hadron duality.

The central value in Eq.(17) is lower by 2% than the one given in Ref.[1] basically due to two reasons: (i) A value of about 4.51 GeV had been used in Ref.[1] for the low scale *kinetic* b quark mass, i.e. a number lower by about 1% than what now the best available determinations yield. (ii) The non-BLM part of the $\mathcal{O}(\alpha_S^2)$ contributions slightly increase the width. The final digit in the central value has to be taken with a grain of salt for the moment while further checks are being performed.

Adding the theoretical uncertainties *linearly* rather than quadratically one arrives at

$$\delta_{theor}|V(cb)|_{\Gamma_{SL}(B)} \simeq 6\% \quad \text{at present} \tag{18}$$

I view such an error estimate as prudent rather than conservative. For there is reason for concern that our determinations of $m_b - m_c$ and of m_b suffer from systematic short-comings.

- Using Eq.(5) relies on an expansion in powers of $1/m_c$ to be numerically reliable. Yet we cannot rest assured of that. For the charm quark is not truly much larger than typical hadronic scales. Eq.(11) implies those scales are around $\sqrt{\mu_\pi^2} \sim 0.7$ GeV making an expansion in powers of $1/m_c$ meaningful.

  However this is not an ironclad conclusion. For it is not ruled out that the relevant hadronic scales are higher; after all we have only a lower bound on μ_π^2. It is conceivable that both the central value of and the uncertainty in $m_b - m_c$ might differ significantly from the values stated above. Combining Eqs.(5,4) leads to an uncomfortably small value of the charm quark mass. The value of $m_b - m_c$ thus contains a central theoretical uncertainty in evaluating $\Gamma(B \to l\nu X_c)$.

- The difference $m_b - m_c$ can be extracted from moments of semileptonic decay spectra through methods that do not suffer the same dependance on $1/m_c$.

- All existing studies determine m_b in very much the same way from Υ spectroscopy. More confidence in its value would be gained through extracting it by a different method.

The error estimate of Eq.(13) had been criticised in the past as unrealistically optimistic by authors that used pole masses or running masses evaluated at the high scale m_b. It was argued that the m_b^5 term suffers from a $\sim 10\%$ uncertainty as do the radiative corrections; this would suggest an irreducible $\sim 20\%$ theoretical uncertainty in the expression for the width. The point missed in these claims was the fact that these two sources of uncertainties are highly correlated as explained above.

I think that reducing this error down to

$$\delta_{theor}|V(cb)|_{\Gamma_{SL}(B)} \simeq 2\% \tag{19}$$

is an attainable goal over the next several years through refining our theoretical tools and calibrating them through experimental cross checks.

However, I see it as quite unlikely that the uncertainties in m_b and $m_b - m_c$ can be reduced to the desired level by theoretical means alone: a *systematically* different way for determining m_b and $m_b - m_c$ can be obtained through a careful analysis of *moments* of semileptonic decay *spectra*. The necessary theoretical tool do exist and require just some polishing; the data can be obtained.

3.1 Criticism in the Literature

The OPE treatment of the semileptonic decays of beauty hadrons has been criticised by Jin in several papers summarized in [10] where he claims that

1. his ansatz is no less based on QCD than the OPE description;
2. the OPE treatment misses relevant kinematical features;
3. his reduced semileptonic width – i.e. where the KM parameter has been factored out – is larger than the OPE reduced width by a significant amount:
$$\frac{1}{|V(qb)|^2}\Gamma^{\mathrm{Ref.}[10]}(B \to l\nu X_q) > \frac{1}{|V(qb)|^2}\Gamma^{\mathrm{OPE}}(B \to l\nu X_q) \; ; \tag{20}$$
this would yield a lower value of $|V(qb)|$ obtained from the same data.

While the last claim is factually correct the first two are not, which makes the last one irrelevant.

Observable spectra are described through folding quark spectra with a (ligh cone) quark distribution function. The actual choice of such a wave function which corresponds to *leading*-twist dynamics affects *spectra*; it *cannot*, however, have an impact on *integrated* rates as we are discussing here.

In addition – and probably most to the point – his description is not only different, but inconsistent as expanding his expression for the semileptonic width reveals:
$$\Gamma^{\mathrm{Ref.}[10]}(B \to l\nu X) \;=\; \Gamma(b \to l\nu q)\cdot\left[1 + \frac{35}{6}\frac{\mu_\pi^2}{m_b^2} + ...\right] \tag{21}$$
$$\Gamma^{\mathrm{OPE}}(B \to l\nu X) \;=\; \Gamma(b \to l\nu q)\cdot\left[1 - \frac{1}{2}\frac{\mu_\pi^2}{m_b^2} + ...\right] \tag{22}$$
Note the difference in sign and magnitude – $\frac{35}{6}/\frac{1}{2} \simeq 12$ – of the μ_π^2/m_b^2 terms in the two expressions. Without any calculation one can immediately see that the OPE result is correct and the other wrong. Since $\mu_\pi^2 = \langle|\vec{p}_b|^2\rangle \simeq \langle m_b^2|\vec{v}_b|^2\rangle$ one realizes that the expression
$$1 - \frac{1}{2}\frac{\mu_\pi^2}{m_b^2} \simeq 1 - \frac{1}{2}|\vec{v}_b|^2 \simeq \sqrt{1-|\vec{v}_b|^2} \tag{23}$$
is nothing but the Lorentz time dilation factor! This kinematical effect has to be present. The expression of Ref. [10] fails completely to reproduce the limiting case of a freely moving b quark. This makes no sense in particular for an ansatz based on the parton model.

Lastly the 't Hooft model is QCD in 1+1 dimensions with N_C, the number of colours, going to infinity. It retains many dynamical features of real QCD like confinement etc., yet can be solved exactly. One finds that the semileptonic $b \to c$ as well as $b \to u$ widths indeed match the OPE results [11].

The question of quark versus hadron kinematics in HQE has been carefully studied since 1993 in several dedicated papers [12, 13, 4, 3] and others more. It has been demonstrated how the full kinematical range can be covered, how the quark phase space and the OPE nonperturbative corrections emerge from the combination of hadronic phase space and bound state effects.

4 $B \to l\nu D^*$ at Zero Recoil

4.1 General Idea

Having measured the lepton spectrum in $B \to l\nu D^*$ one can extrapolate to the point of zero recoil to extract $|V(cb)F_{D^*}(0)|$. This extrapolation will introduce a theoretical uncertainty about which I have nothing new to add.

Heavy quark symmetry tells us that

$$\lim F_{D^*}(0) = 1 \quad \text{as} \quad m_b \to \infty \tag{24}$$

must hold; i.e., we have

$$\begin{aligned} F_{D^*}(0) &= 1 + \mathcal{O}(\alpha_S/\pi) + \delta^A_{1/m^2} + \delta^A_{1/m^3} + ... \\ \delta^A_{1/m^2} &= \mathcal{O}(1/m_c^2) + \mathcal{O}(1/m_c m_b) + \mathcal{O}(1/m_b^2) \end{aligned} \tag{25}$$

The theoretical challenge consists of calculating the perturbative as well as nonperturbative corrections to this exclusive process.

The absence of nonperturbative corrections of order $1/m_{b,c}$ was first noted by Shifman and Voloshin [15]. It was later analyzed in more detailed by Luke [16] and is often referred to as Luke's theorem.

Calculating the perturbative corrections provides us with a technical challenge where the last word has not been spoken yet; however they do not pose a conceptual problem. The conceptual challenge has come from the power corrections.

Till 1994 the canonical claim had been that HQET allows to determine these contributions reliably and that they are quite small

$$\delta^A_{1/m^2} = (-2 \pm 1)\% \tag{26}$$

leading to $F_{D^*}(0) \simeq 0.97$.

In 1994 the OPE-based heavy quark expansion was applied to this problem [4, 14]. Employing a (zero velocity) sum rule the analysis yielded $F_{D^*}(0) \simeq 0.9$. This value has now been widely adopted as the central value.

4.2 Power Corrections

Heavy quark expansions allow to calculate the inclusive width for all channels generated by the axial current: $B \xrightarrow{j^5_\mu} X_A$. The hadronic final state X_A has three components, namely

- D^* as the lowest state;
- higher excitations like D^{**}, etc. and
- a quasi-continuum formed by $D + \pi's$ configurations where the distinction between this and the preceding component is fluid.

Turning the argument around $B \to D^*$ is given as the difference between the inclusive zero recoil width $B \to X_A$ and the higher excitations including the continuum. The formfactor F_{D^*} can thus be expressed through three types of contributions:

1. those coming from local operators emerging in a $1/m_Q$ expansion, as characterized by μ_π^2, μ_G^2 and terms of order $1/m_Q^3$ (and higher). One should note here that there are three variants of $1/m^2$ terms, namely $1/m_c^2$, $1/(m_c m_b)$ and $1/m_b^2$. Obviously the first one will usually dominate.
2. *nonlocal* contributions from higher excitations;
3. perturbative ones.

Combining them I obtain

$$\begin{aligned} F_{D^*}(0) &\simeq 0.89 - 0.026 \cdot \frac{\mu_\pi^2 - 0.5\,\mathrm{GeV}^2}{0.2\,\mathrm{GeV}^2} \pm 0.02|_{excit} \pm 0.01|_{pert} \pm 0.025_{1/m^3} \\ &\simeq 0.89 \pm 0.08|_{theor} \end{aligned} \tag{27}$$

The central value has been lowered since a more careful analysis of the perturbative corrections that partially accounts for $1/m_Q^3$ terms suggests a smaller short distance renormalization factor [21].

A few comments will help to gain a proper perspective of these findings:

- One should note that the impact of μ_π^2 on $F_{D^*}(0)$ and on $\Gamma_{SL}(B)$ is *anticorrelated*: while a larger value of μ_π^2 *reduces* $F_{D^*}(0)$ and thus leads to a *larger* value of $|V(cb)|$ to be obtained from the observable $|F_{D^*}(0)V(cb)|$, it *enhances* $\Gamma_{SL}(B)$ through a larger value for $m_b - m_c$ and thus *reduces* $|V(cb)|_{\Gamma_{SL}(B)}$, see Eqs.(17,27).
- The value of μ_π^2 represents a major common source of the uncertainty in both cases; however the origin is very different:
  - In $\Gamma_{SL}(B)$ it mainly reflects how we infer the value of $m_b - m_c$. Once the latter has been determined differently, namely from spectra in semileptonic decays, the impact of μ_π^2 will largely fade as far as $\Gamma_{SL}(B)$ is concerned.

  - For $F_{D^*}(0)$ on the other hand μ_π^2 forms an essential part of our evaluation.

- The leading nonperturbative corrections to $F_{D^*}(0)$ are controlled by a $1/m_c^2$ term. A deviation of $F_{D^*}(0)$ from unity by about 10% is then much more reasonable then the originally claimed 2%. Yet this raises also the legitimate concern whether such an expansion is numerically reliable because of the charm quark mass being only moderately large. While I have expressed this worry also in the discussion of the total semileptonic width, it is much more serious here:

  - The $1/m_c$ expansion enters the treatment of $\Gamma_{SL}(B)$ only in a secondary way, namely as a tool to determine $m_b - m_c$. As stated before, that mass difference can be extracted from spectra in semileptonic B decays in ways that do *not* depend on a $1/m_c$ expansion.
  - In $F_{D^*}(0)$ on the other hand the $1/m_c$ expansion forms an integral part of the analysis and no good way has been found to determine these power corrections in a more reliable way.

- Estimates of the uncertainties in the $1/m^3$ corrections and of the contributions from the higher excitations do not have a firm foundation yet.

- At present I see little basis for adding the theoretical uncertainties in quadrature.

Some moderate reduction in some of the uncertainties should be achievable, in particular concerning μ_π^2; modelling the excitations might also provide some useful handles. Yet in contrast to the situation with the total semileptonic B width I do not see how the theoretical uncertainty in $F_{D^*}(0)$ can be reduced significantly, i.e. be cut in half. For that to come about a genuine breakthrough had to happen.

5 Using $\tau(b)$ & BR$(b \to l\nu X)$

Rather than determining the lifetimes and semileptonic branching ratios of *specific* beauty hadrons one can measure these observables *averaged* over the beauty hadrons present in the sample. This pedestrian implementation of quark-hadron duality will certainly minimize the statistical errors. The question is how great a price one pays in systematic uncertainties.

The average semileptonic branching ratio of beauty hadrons in a given sample is expressed by

$$\langle BR_{SL}(b)\rangle = w_{B_d}\frac{\Gamma_{SL}(B_d)}{\Gamma(B_d)} + w_{B^+}\frac{\Gamma_{SL}(B^+)}{\Gamma(B^+)} + w_{B_s}\frac{\Gamma_{SL}(B_s)}{\Gamma(B_s)} + w_{\Lambda_b}\frac{\Gamma_{SL}(\Lambda_b)}{\Gamma(\Lambda_b)} \tag{28}$$

with the w_{H_b} denoting the relative weights of the various beauty hadrons in the data sample under study; Λ_b is used in a generic way to include other beauty baryons $\Xi_b^{0,-}$.

One predicts that the semileptonic widths of the beauty *mesons* differ by less than a percent

$$\Gamma_{SL}(B_d) \simeq \Gamma_{SL}(B^+) \simeq \Gamma_{SL}(B_s) \equiv \Gamma_{SL}(B) \tag{29}$$

Using the constraint

$$w_{B_d} + w_{B^+} + w_{B_s} + w_{\Lambda_b} = 1 \tag{30}$$

one arrives at

$$\langle BR_{SL}(b)\rangle \simeq \Gamma_{SL}(B)\langle\tau(b)\rangle \left[1 + w_{\Lambda_b} \cdot \frac{\tau(\Lambda_b)}{\langle\tau(b)\rangle}\left(\frac{\Gamma_{SL}(\Lambda_b)}{\Gamma_{SL}(B)} - 1\right)\right] \tag{31}$$

where $\langle\tau(b)\rangle$ denotes the average lifetime of beauty hadrons in the sample. It has been measured with considerable accuracy at LEP where the 'beauty cocktail' is described by

$$w_{B^+} = w_{B_d} = 0.395^{+0.013}_{-0.014}\,,\; w_{B_s} = 0.108 \pm 0.014\,,\; w_{\Lambda_b} = 0.102^{+0.023}_{-0.021} \tag{32}$$

I do not claim a quantitative understanding of beauty baryon lifetimes – nor do I need to. For the second factor in the square bracket will not amount to more than a one percent effect or so, since $w_{\Lambda_b} \simeq 0.1 \pm 0.02$ and $|\Gamma_{SL}(\Lambda_b)/\Gamma_{SL}(B) - 1| \leq 0.2$ represents a generous bound for *semileptonic* widths.

The measurement of the average semileptonic branching ratio and beauty lifetime thus amounts to a measurement of $\Gamma_{SL}(B)$ with *no* additonal uncertainties of numerical significance. The same is likewise true for $\langle BR(b \to l\nu X_u)\rangle$ and $\Gamma(B \to l\nu X_u)$.

6 Theoretical Uncertainty in $|V(ub)/V(cb)|$

6.1 $|V(ub)|$

$|V(ub)|$ can be determined from the measured value for $\Gamma(B \to l\nu X_u)$ again using HQE technology (see [6] for a summary):

$$\begin{aligned} |V(ub)|_{B\to l\nu X_u} = 0.00442 \quad &\cdot \quad \left(\frac{\mathrm{BR}(B \to l\nu X_u)}{0.002}\right)^{1/2}\left(\frac{1.55\ \mathrm{ps}}{\tau_B}\right)^{1/2} \\ &\cdot \quad \left(1 \pm 0.01|_{pert} \pm 0.018|_{1/m_b^3} \pm 0.035|_{m_b}\right) \end{aligned} \tag{33}$$

As far as the angles of the KM unitarity triangle are concerned which control CP asymmetries in B decays, one is interested more in $|V(ub)/V(cb)|$ than $|V(ub)|$ itself. It is then quite relevant to analyze whether some theoretical uncertainties drop out from this ratio.

6.2 $|V(ub)/V(cb)|$

Most of the theoretical uncertainties in $|V(cb)|_{\Gamma_{SL}(B)}$ and $|V(ub)|_{B\to l\nu X_u}$, Eqs.(17, 33), are *independant* or only mildly correlated:

- Whereas the value of μ_π^2 has great impact on $|V(cb)|_{\Gamma_{SL}(B)}$ as yardstick for $m_b - m_c$, it affects $|V(ub)|_{B\to l\nu X_u}$ very little.
- The probably leading effect in $\mathcal{O}(1/m_b^3)$, namely the weak annihilation contribution to semileptonic B^+ decays, does not affect $|V(cb)|_{\Gamma_{SL}(B)}$.
- While a priori the perturbative uncertainties could be highly correlated for $b \to c$ and $b \to u$, it turns out to be otherwise.

I will add the $1/m_b^3$ uncertainty in $|V(ub)|_{B\to l\nu X_u}$ and $|V(cb)|_{\Gamma_{SL}(B)}$ in quadrature; likewise for the perturbative one.

One uncertainty is significantly correlated:

- The value of $m_b(1\text{ GeV})$ is obviously an important input in both cases, although less so in $b \to c$ where $\Gamma(B \to l\nu X_c) \propto (m_b - m_c)^{5-p} m_b^p$ with $p \sim 2$ than in $b \to u$ with $\Gamma(B \to l\nu X_u) \propto m_b^5$.

Hence I infer the following theoretical uncertainties for $|V(ub)|_{B\to l\nu X_u}/|V(cb)|_{\Gamma_{SL}(B)}$:

$$\delta_{theor}(|V(ub)|_{B\to l\nu X_u}/|V(cb)|_{\Gamma_{SL}(B)}) = \pm 0.02|_{m_b} \pm 0.025|_{\mu_\pi^2} \pm 0.018|_{pert} \pm 0.03|_{1/m_b^3} \quad (34)$$

Adding these errors linearly might be overly conservative; adding them *in quadrature* one arrives at an overall uncertainty of 4 - 5%.

The situation with respect to $|V(ub)|_{B\to l\nu X_u}/|V(cb)|_{B\to D^*}$ is simpler in the sense that there are hardly any correlations in the theoretical uncertainties. Adopting the same strategy as just sketched, I estimate:

$$\delta_{theor}(|V(ub)|_{B\to l\nu X_u}/|V(cb)|_{B\to D^*}) = 0.025|_{\mu_\pi^2} \pm 0.02|_{excit} \pm 0.035|_{m_b} \pm 0.04|_{1/m_b^3} \pm 0.015|_{pert} \quad (35)$$

representing an overall uncertainty of 6 - 7 % when added in quadrature (adding them linearly might be even less reasonable than for Eq.(34)).

7 Summary and Conclusions

The significant conclusions from this discussion can be summarized as follows:

- The KM parameter $|V(cb)|$ can at present be extracted
  - from the total semileptonic width,
  - from $B \to l\nu D^*$ at zero recoil and
  - from the 'average' beauty semileptonic branching ratio and lifetime

  with about a 6% *theoretical* accuracy.

- $|V(ub)|$ can be obtained from $\Gamma(B \to l\nu X_u)$ or $\langle \mathrm{BR}(b \to l\nu u)\rangle$ in conjunction with $\langle \tau(b) \rangle$ with about a 7-8 % theoretical accuracy.

- The ratio $|V(ub)/V(cb)|$ can the be determined with an uncertainty of better than 5%.

- The very attractive fundamental feature of Heavy Quark Theory that many different obserables are related to each other – sometimes in subtle ways – means that there are numerous correlations and that any change in one of the basic quantities implies changes in several observables. This has to be taken into account.

- Presently it is prudent to combine theoretical errors *linearly* as it was mostly done above: theoretical errors in general do not follow a Gaussian distribution; correlations are not always manifest; expansions in $1/m_c$ might not be numerically reliable; the value of the crucial quantity m_b has so far been inferred from a single reaction only, etc.

- This problem can be overcome, though. Again using the very fact of few quantities controlling several a priori independant observables one will be able – in due time – to extract the values of these quantities in different ways. If these redundant extractions indeed agree one has established a new level of theoretical control and can add also theoretical uncertainties in quadrature.

- We can map out the strategy for reducing the theoretical error in $\Gamma_{SL}(B)$ to 2%. It is hard to see how the same improvement can be achieved for the description of $B \to l\nu D^*$.

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